Shape-Based Visualization for Biomedical Data

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Motivation

Complex biomedical data
Difficult to understand

Shape information is a good clue to understand the complexities of these biomedical data

However, defining shape is not trivial

Solution: Laplace-Beltrami Operator (LBO) for volume data involves heat kernel computation
A Novel way to define shapes

LBO and Heat Kernel

\[ \Sigma \] is a Riemannian manifold defined by a function \( f: \Sigma \rightarrow \mathbb{R} \) with a metric \( g \). The LBO \( \Delta \) of that function is defined as:

\[ \Delta f = \text{div} (\text{grad} f) \]

where \( \text{grad} f \) is gradient of \( f \) and \( \text{div} \) is divergence of manifold.

Discrete LBO for 2D case:

\[ L_{ij} = \frac{\cot \alpha_i + \cot \beta_j}{2A_i} \text{ if } i,j \text{ are adjacent} \]

\[ = \sum_{k \in V(i)} \frac{\cot \alpha_i + \cot \beta_j}{2A_i} \text{ if } i=j \]

\[ = 0 \text{ otherwise} \]

The angles \( \alpha_i \) and \( \beta_j \) opposite to the edge \( ij \), \( j \) and \( k \) are the points at 1-ring neighborhood of \( i \)

The heat kernel is defined as the probability of reaching the point \( y \) from point \( x \) in a time \( t \) using a random path. Heat value at a time \( t \), \( H(t) \) is defined using LBO as follows:

\[ H_i(t) = H_i(t-1) + L_{ij} \]

Results

Steps of mesh construction:
- Octree construction
- K-mean clustering
- Initial mesh construction
- 3D Delaunay Triangulation

• 3 levels of decimation
• Coarser, finer and finest level meshes

Two levels of meshes (coarser and finer) shown for the engine dataset

Two levels of decimation shown for the 2D case for better understanding
(Note: Any number of levels of decimation can be constructed)

Delaunay Mesh Construction

Steps of mesh construction:

- Octree construction
- K-mean clustering
- Initial mesh construction
- 3D Delaunay Triangulation

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Two levels of meshes (coarser and finer) shown for the engine dataset

LBO and Heat Kernel

Sphere data

MRI Head Data

Neghip data

Fuel data

Shape information of neghip protein from different time steps